1. [4%] Solve the 1\textsuperscript{st}-ordered differential equation \( \frac{dy}{dx} = \frac{2x}{e^{x^2+2} + e^{-x^2-x}} \).  

2. [8%] Solve the 2\textsuperscript{nd}-ordered differential equation \( y'' - 3y' + 2y = \cosh x + \sinh 2x \).  

3. [6%] A function is defined by \( f(t) = \begin{cases} t, & 0 \leq t < 2 \\ 2, & 2 \leq t < 4 \\ 2 - t, & 4 \leq t < 6 \\ 0, & \text{elsewhere} \end{cases} \). Please carefully sketch \( f(t) \) and to find its Laplace transform \( F(s) \).  

4. Use the Laplace transform to solve the following equations:  
   (a) [6%] \( f(t) = \cos(2t) + \int_0^t f(\tau)e^{2(t-\tau)}d\tau \)  
   (b) [6%] \( f(t) = e^{-t} \cos t + \int_0^t f(t-\tau)d\tau \)  
   (c) [4%] \( h'' - 5h' + 6h = \delta(t), \ h(0) = 0, \ h'(0) = 0 \)  
   (d) [8%] \( y'' - 5y' + 6y = \cos t, \ y(0) = 0, \ y'(0) = 0 \)  

5. [8%] Verify the response of problem 4d is the convolution \( y(t) = \cos(t) * h(t) \) of \( \cos(t) \) with \( h(t) \), where \( h(t) \) is the response of problem 4c.  

6. [10%] An inner product defined on the vector space \( P_2 \) of all polynomials of degree less than or equal to 2, is defined by \( (p, q) = \int_{-1}^1 p(x)q(x)dx \). Use the Gram-Schmidt orthogonalization process to transform the basis \( B = \{x^2-x, \ x^2+1, \ 1-x^2\} \) for \( P_2 \) into an orthonormal basis \( C \).  

7. [10%] Consider the matrix \( A = \begin{pmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{pmatrix} \). Find an orthogonal matrix \( P \) that diagonalizes \( A \) and the diagonal matrix \( D \) such that \( D = P^*AP \).  

8. [10%] Find the work done by the force \( F(x, y, z) = (4x^2 - y^3)i + (x^3 + y^2)j \) around the closed curve indicated by arrows in the following figure.
\[ \mathbf{u} \text{ is the unit vector normal to } S \text{ and oriented outward.} \]

\[ \mathbf{b} = \mathbf{g}(\mathbf{r} \cdot \mathbf{a}) \int \int \int \]

Theorem to prove the Cauchy's Law: \[ \mathbf{b} = \nabla \times (\mathbf{u} \cdot \mathbf{a}) \int \int \int \]

\[ \text{Inverse square field } \mathbf{b} = \mathbf{g} \text{ where } \mathbf{b} = \mathbf{a} \text{ is the position vector of } (x', y', z'). \]

\[ \text{Field at a point charge } b \text{ located at the origin is given by the} \]

\[ 10. \quad \text{The electric field at a point } (x', y', z') \text{ of a charged sphere is inside the} \]

\[ \text{cylinder } z = r_x \text{ and } z = r_y \text{ is the unit vector normal to } S \text{ and oriented} \]

\[ \text{of the surface integral.} \]

9. \[ \text{Evaluate the surface integral.} \]

\[ x = x' + x' \]

\[ y = y' + y' \]

\[ z = z' + z' \]