1. The pair of BPSK signals $s_i(t)$ and $s_o(t)$ used to represent binary symbols 1 and 0 in terms of one basis function

$$
\phi(t) = \frac{2}{\sqrt{T_b}} \cos(2\pi f_c t)
$$

as follows: $s_i(t) = \sqrt{E_b} \phi(t), \quad 0 \leq t \leq T_b$ and $s_o(t) = -\sqrt{E_b} \phi(t), \quad 0 \leq t \leq T_b$ where $E_b$ is the transmitted signals energy per bit. Let the carrier frequency $f_c$ is chosen equal to integer multiples of $1/T_b$. The received signal:

$$
x(t) = s_i(t) + w(t), \quad 0 \leq t \leq T, \ i = 1, 2
$$

where $w(t)$ is a white Gaussian noise process of zero mean and power spectral density $\frac{N_0}{2}$. Assuming the BPSK signal is applied to a correlator supplied with a phase reference that lies within $\theta$ radians of the exact carrier phase. **Determine the effect of the phase error $\theta$ on the average probability of error of the system.** 10%

2. Consider a cosine wave of frequency $f_c$, and amplitude $A_n$, applied to a delta modulator of step-size $\Delta$ with sampling period $T_s$. Assuming a load of $1\Omega$, **what** is the maximum power that may be transmitted without slope-overload distortion? 10%

3. Consider a white Gaussian noise of zero mean and power spectral density of $N_0/2$, which is passed through an ideal band-pass filter of passband magnitude response equal to two, midband frequency $f_c$, and bandwidth $2B$ as shown in Figure 1. **Find the autocorrelation functions of (1). The filtered noise $n(t)$** 10% **and (2) its in-phase and quadrature components.**

![Figure 1](image)

4. A voice signal (300 to 3300 Hz) is digitized such that the equalization distortion $\leq \pm 0.1\%$ of the peak-to-peak signal voltage. 

Assume a sampling rate of 8000 samples/s and a multilevel PAM waveform with $M=64$ levels. **Find the theoretical minimum system bandwidth that avoids ISI.** 10%

5. A random process $X(t)$ is defined by: $X(t) = A \cos(2\pi f_c t)$ where $A$ is a Gaussian-distributed random variable of zero mean and variance $\sigma^2$. This random process is applied to an ideal integrator, producing the output $Y(t) = \int X(t)dt$

(a) **Determine** the probability density function of the output $Y(t)$ at a particular time $t_i$. 10%

(b) **Determine** whether or not $Y(t)$ is ergodic (should explain the reason). 5%

6. An FM signal with a frequency deviation of 20 kHz at a modulation frequency of 10 kHz is applied to two frequency multipliers connected in cascade. The first multiplier triples the frequency and the second multiplier doubles the frequency. **Determine** (1) the modulation index obtained at the second multiplier output, 6% and (2) the frequency separation of the adjacent side frequencies of this FM signal at the second multiplier output? 4%
7. Consider the signal \( s(t) = \begin{cases} 
A, & 0 \leq t \leq T/4 \\
-A/2, & T/4 \leq t \leq 3T/4 \\
A, & 3T/4 \leq t \leq T \\
0, & \text{otherwise} 
\end{cases} \). (a) Write and sketch the matched filter of this signal. (b) What is the peak value at the output of the matched filter? (c) The mathematical operation of a matched filter is convolution; a signal is convolved with the impulse response of a filter. However, the mathematical operation of a correlator is correlation; a signal is correlated with a replica of itself. Explain the reason why the term “matched filter” is often used synonymously with “correlator”. 10\% = 3\% + 3\% + 4\%.

8. Consider a binary source for which symbol 0 occurs with probability \( p_0 \) and symbol 1 with probability \( p_1 = 1 - p_0 \). Assume that the source is memoryless so that successive symbols emitted by the source are statistically independent. Prove that the entropy is maximized when the probabilities \( p_0 \) and \( p_1 \) are equal 10\%.

9. (a) Show that the three functions illustrated in Figure 2 are pairwise orthogonal over the interval \((-2,2)\). 6\%
(b) Determine the value of the constant, \( A \), that makes the functions in part (a) an orthonormal set. 4\%