1. Please solve the following questions.
   (a) Find a complete solution of $y'' - y = \cos x$. (10%)  
   (b) Using the method of variation of parameters, show that the complete solution of $y'' + ky = f(x)$  
       is $y(x) = A \cos kx + B \sin kx + \frac{1}{k} \int_0^x \sin k(x-s)f(s)ds$. (10%)  

2. The linear time-invariant system with unit impulse response $h(t) = \sum_{n=0}^{\infty} \delta(t-4n)$ and input $x(t) = \begin{cases} \sin(t), & 0 \leq t < 1 \\ 0, & \text{elsewhere} \end{cases}$ shown in Fig. 1.  
   (a) Evaluate and sketch the system output $y(t)$. (10%)  
   (b) Use the Laplace transform to check your answer of question 2a is correct. (10%)  

   ![Fig. 1 Linear time-invariant system](image)  

3. For each suitable function $f(t)$, $t \geq 0$. Please solve the following questions.  
   (a) With $s = \sigma + j\omega$, show that the Laplace transform of $f(t)$ equals to the Fourier transform of $f(t)e^{-\sigma t}$. (35%)  
   (b) Let $f(t) = t^r$, $t \geq 0$, show your answer of question 3a is correct. (5%)  

4. (a) Show that if $0 < \theta < \pi$, then $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ has no real eigenvalues and consequently no real eigenvectors. (10%)  
   (b) Give a geometric explanation of the result in 4a. (10%)  

5. Let $x_1$, $x_2$, and $x_3$ be distinct real numbers such that $x_1 < x_2 < x_3$, and let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the function defined by the formula $T(p(x)) = \begin{bmatrix} p(x_1) \\ p(x_2) \\ p(x_3) \end{bmatrix}$.  
   (a) Show that $T$ is a linear transformation. (10%)  
   (b) Show that $T$ is one-to-one. (10%)  

6. Let $W$ be the space spanned by $f = \sin x$ and $g = \cos x$. Show that for any value of $\theta$, $f_1 = \sin(x+\theta)$ and  
   $g_1 = \cos(x+\theta)$ form a basis for $W$. (10%)