1. Solve the following differential equations:

   (a) \[ xy' + y = (xy)^2 \quad (7\%) \]

   (b) \[ y'' + 4y' + 13y = \frac{1}{3} e^{-2t} \sin 3t \quad \text{y(0)} = 1, \quad y'(0) = -2 \quad (8\%) \]

2. (a) Find the inverse Laplace transform of the function \( F(s) = \frac{b}{s^3(s + a)} \); \( a, b \) are positive real numbers. (7%)

   (b) Solve the following differential-integral equation. (Hint: Applying Laplace transform and convolution theorem)
\[
y'(t) + y(t) - \int_0^t y(v) \sin(t - v)dv = -\sin t \quad \text{y(0)} = 1 \quad (8%)\]

3. (a) Find the Fourier series in real form of the following function (9%)
\[
f(x) = \begin{cases} 3x & , -2 < x < 2, \quad f(x + 4n) = f(x) \\
\end{cases}
\]

   (b) Find the Fourier series in complex form of the following function (6%)
\[
f(x) = e^{2x} \quad , -\pi < x < \pi \quad f(x + 2n\pi) = f(x)
\]

4. (a) If \( f(z) \) is analytic in a simply connected domain \( D \), then for every simple closed path \( C \) in \( D \), show that \( \oint_C f(z)dz = 0 \) using Green's theorem and Cauchy–Riemann Equations. (8%)

   (b) Calculate the complex line integral \( \oint_C \frac{1}{z^2 - 1}dz \), using residue theorem.

   where \( C \): (1) \( |z| = \frac{1}{2} \), (2) \( |z - 1| = 1 \), (3) \( |z + 1| = 1 \), (4) \( |z| = 2 \) (12%)
5. (a) Suppose \( T : R^2 \rightarrow R^2 \) is a linear transformation and that \( T(1,0) = (1,4) \), \( T(1,1) = (2,5) \).

(1) What is \( T(3,5) \) ? (6%)

(2) What is the null space of \( T \) ? (3%)

(3) What is the range of \( T \) ? (3%)

\[
\begin{aligned}
&x_{k+1} = \frac{1}{2} x_k + y_k \\
y_{k+1} = \frac{1}{16} x_k + \frac{1}{2} y_k
\end{aligned}
\quad (k = 0, 1, 2, \ldots)
\]

and \( x_0 = 0, y_0 = 1 \), find \( x_n \) and \( y_n \) (13%)

(b) Let

6. Let \( \vec{F} = xy\vec{i} + yz\vec{j} + xz\vec{k} \), calculate surface integral \( \iint_S \vec{F} \cdot \vec{n} dA \) using Gauss divergence theorem, where \( S \) is the bounding surface (with outer unit normal \( \vec{n} \)) of the unit cube by \( 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1 \). (10%)