1. Solve the following differential equations: (24 %)
   (a) \( 1 - xy' = 0 \)
   (b) \((y + x^2)dx + (x + 2y)dy = 0, \) \(y(0) = 1\)
   (c) \( y' + y \tan x = 0 \)
   (d) \( y'' - 3y' + 2y = e^{3x} \)
   (e) \( x^2y'' - xy' + 5y = 0 \)
   (f) \( y'' + 2y + 10y = 0 \)

2. Find the general solution for a given higher order differential equation with its characteristic equation is
\[
(\lambda^3 + 6\lambda^2 + 11\lambda + 6)(\lambda^2 - 4)(\lambda^2 + 2\lambda + 5)^2 = 0
\]
(for example, the general solution for \( \lambda^2 - 1 = 0 \) is \( y = c_1e^x + c_2e^{-x} \)) (8 %)

3. Find the Maclaurin series of \( f(x) = \ln(1 + x) \) (10 %)

4. Find the Laplace transform of the following functions: (10 %)
   (a) \( e^t \cos(4t) \)
   (b) \( t^2 \sin t \)

5. Find the directional derivative of \( f(x, y, z) = x^2 - 3y + 4z \) at \((1,2,3)\) in the direction of \((2,-1,-1)\). (10 %)

6. Find the eigenvalues and the corresponding eigenvectors of the matrix \( A \). (10 %)
\[
A = \begin{bmatrix}
-2 & 2 & -3 \\
2 & 1 & -6 \\
-1 & -2 & 0
\end{bmatrix}
\]

7. \( \vec{p} = (1,1,1), \) \( \vec{q} = (2,-1,-4), \) \( \vec{r} = (-1,2,0) \), find (a) the angle between \( \vec{p} \) and \( (\vec{r}+2\vec{q}) \) (b) the volume of a tetrahedron which is determined by three edge vectors \( \vec{p}, \vec{q}, \) and \( \vec{r} \). (12 %)

8. Find the straight line \( L_1 \) through the point \((1,3)\) in the xy-plane and perpendicular to the straight line \( x-2y+2=0 \) (8 %)

9. Solve \[
\begin{bmatrix}
1 & -1 & -1 \\
2 & 3 & 5 \\
1 & -2 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
-3 \\
7 \\
-11
\end{bmatrix}
\]
by Cramer's rule. (8 %)