1. Solve the following differential equations: (40 pts; (a)-(d), 4 pts each; (e)-(h), 6 pts each)
   (a) \((2y + 1)y' - x^3 = 0\)
   (b) \((y^2 + 2x)ax + (3xy^2 + 1)dy = 0\)
   (c) \((x^2 y + y)dx + xdy = 0\)
   (d) \(y^n + y'^n + y = 0\)
   (e) \(y^n + 6y'^n + 9y = 18\cos 3x\)
   (f) \(y^n + x^n y = x^n y^n\)
   (g) \(\delta^2 u / \delta x^2 y = x - y\)
   (h) \(y^n + y = 4 \cdot 5^n(t - 2\pi)\)

2. (a) Find the Fourier series of \(f(x) = \sin x\), \(-\pi < x < \pi\) (10 pts)
   (b) Compute \(\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}\) (4 pts)

3. Find the rank of a given matrix \([A]\). (6 pts)
\[
A = \begin{bmatrix}
1 & 0 & 3 & 0 \\
0 & 2 & 4 & 6 \\
3 & 0 & 5 & 2 \\
2 & 3 & 0 & 1
\end{bmatrix}
\]

4. Find the Laplace transforms of the following functions: (8 pts)
   (a) \(\int_0^t u^2 e^{\nu} du\)
   (b) \(\sin(2t + 3)\)

5. Find the inverse Laplace transforms of the following functions: (8 pts)
   (a) \(\frac{1}{s(s^2 + 16)}\)
   (b) \(\frac{\pi(2s - 3)}{s^2 + \pi^2}\)

6. Find the matrix of \([C]\), satisfy \([C][A] = [B]\) with \(A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}\) and \(B = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}\) (8 pts)

7. Find the angle between \(5x + 3y - z = 3\) and \(4x - y + 2z = -1\). (8 pts)

8. Find the interval of convergence for \(\sum_{k=0}^{\infty} \frac{(4x - 5)^k}{3^{k+1}}\) (8 pts)

(Hint) You may use \(\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta\) in solving problem 2 & problem 4.