1. Let a first-order differential equation be given by \( x - xy - y' = 0 \). (a) Show that \( \mu(x) = e^{x/2} \) is an integrating factor of the differential equation; (b) Use the result in (a) to find the general solution.

2. Find the general solution of \( y' = \frac{3x - y + 9}{x + y - 1} \).

3. Let a second-order differential equation be given by \( y'' + 2y' + y = 0 \). (a) Show that \( y(x) = e^x \) is a solution; (b) Use the result in (a) to find the second solution by reduction of order.

4. Solve \( x^2 y'' + 2xy' + y = x \).

5. Let \( A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \). Find its determinant, rank, and eigenvalues.

6. Use the Laplace transform to solve \( y' + 4y = f(t); \ y(0) = 0 \), in which \( f(t) = \begin{cases} 0, & t < 3 \\ 2, & t \geq 3 \end{cases} \).

7. Let \( A = \begin{bmatrix} 12 & 5 & 5 & 5 & 5 \\ 5 & 12 & 5 & 5 & 5 \\ 5 & 5 & 12 & 5 & 5 \\ 5 & 5 & 5 & 12 & 5 \\ 5 & 5 & 5 & 5 & 12 \end{bmatrix} \). Find the inverse of the 5\times5 matrix \( A \).

8. Let \( \phi(x, y, z) = \sin(xy) \). Compute \( \nabla \phi \) and verify that \( \nabla \times (\nabla \phi) = \mathbf{0} \).

9. Evaluate \( \iint_S zd\sigma \) if \( S \) is the part of the plane \( x + y + z = 4 \) lying above the rectangular \( 0 \leq x \leq 1, 0 \leq y \leq 2 \).

10. Let \( f(x) = \begin{cases} -1, & -\pi \leq x \leq 0 \\ 1, & 0 < x \leq \pi \end{cases} \). Find the Fourier series of the function \( f(x) \).